Connected Regions, 18. Polynomial Expansions and Conformal Maps, 19. Univalent Functions.

The study of applied mathematics and computation using tools of complex analysis provides an indispensable insight and understanding of not only when, why, and how well the methods of these subjects work, but also for deriving new methods. Unfortunately, an examination of the curriculum of most schools, or an examination of textbooks on complex variables, shows that the importance of this understanding is sadly underemphasized. Henrici's three volumes provide this understanding, and thus they fill a dire need in our educational system. The books are excellent reference books for applied mathematicians, engineers and physicists, and they may be used as textbooks for good students. Exercises, problems, and seminar topics appear at the end of chapters.

Henrici's Volumes 1 and 2 have been reviewed in *Math. Comp.*, v. 31, 1977, pp. 325–326, MR **51** #8378, and MR **56** #12235. The present reviewer wholeheartedly agrees with the praiseworthy remarks in these reviews.

Volume 3 shares features of Volumes 1 and 2, in that it contains exciting presentations which can be found in no other textbooks. Chapter 13 is a new, self-contained and well-written presentation of the DFT method. Here one finds an excellent exposition of the discrete Fourier transform method for Fourier, power series and integrals, numerical harmonic analysis for Fourier series and integrals, evaluation of coefficients of Laurent series, residues and zeros, the conjugate periodic functions, convolutions, and multivariate DFT. The material of Chapter 14, which was previously available only in technical monograph translations from Russian, provides powerful methods for solving problems of potential theory. Included in this chapter are methods of solution of the Riemann and Privalov problems for both closed and open arcs, as well as the recently developed Burniston-Siewert method for solving transcendental equations. Chapter 15 discusses a variety of methods of solution of Dirichlet and Neumann problems for planar potentials. Chapters 16 through 18 discuss conformal mapping, as well as the very exciting and newly developed area of the construction of conformal maps. One finds in these chapters a discussion of the osculation methods for numerical conformal mapping, the integral equations of Symm and Berrut, the mapping methods of Timman, Fornberg and Wegmann, and a formal theory of Faber polynomials and Faber functions. Finally, Chapter 19 presents an elementary and self-contained version of the recently discovered proof of the Bieberbach conjecture in the theory of univalent functions.

Congratulations to Peter Henrici for completing such a monumental, important and exciting work!

F. S.

18[41A50].—G. G. LORENTZ, *Approximation of Functions*, 2nd ed., Chelsea, New York, 1986, ix + 188 pp., $23\frac{1}{2}$ cm. Price $14.95.

Approximation Theory is a well-established part of analysis which, after more than 100 years of activity, remains a vital research area with numerous applications. This branch of mathematics deals with the problem of approximating a complicated (or possibly unknown) function by a relatively simple function (such as a polynomial). There are especially close ties to three other mathematical fields: 1) Fourier Analysis, 2) Numerical Analysis, and 3) Statistical data analysis. Its applications include data fitting, computer evaluation of special functions, numerical solution of differential and integral equations, and computer representation of curves and surfaces (for computer-aided design and manufacture). In addition it remains (sometimes under the alias Constructive Function Theory) an important tool in other branches of mathematics.

The importance of the subject can be judged by the fact that 1) papers on it can be found in numerous journals, including several devoted exclusively to Approximation Theory, 2) there continue to be numerous conferences in a variety of countries on the subject, and 3) over the years a large number of books on the subject have been written, and continue to be written.

The subject of this review is the second edition of a book which first appeared in 1966. Except for some minor corrections (and the insertion of one new result on rational approximation), this second edition is identical with the first, and thus those familiar with the original classic need read no further.

The author's stated aim in writing this book was to provide an accessible but reasonably complete treatment of several topics in Approximation Theory. The book is aimed at the graduate or advanced undergraduate level, and includes a considerable number of problems (some of which are rather more challenging than others). As everyone familiar with the book knows, the author has been very successful at meeting his goals, and the book is indeed a very readable introduction to the subject and makes an excellent textbook.

The heart of the book is contained in Chapters 1–5, which deal with best approximation from a finite-dimensional subspace of a normed linear space. Most of the results concern real functions, and in fact most of the action takes place in $C(A)$, where A is a compact set. In these chapters one may find the classical approximation results on existence of best approximants, uniqueness, characterization, and degree of approximation. Along the way one is treated to a discussion of positive linear operators, moduli of smoothness, classes of smooth functions, Bernstein and Markov inequalities, and a wealth of other tools and techniques. Special attention is devoted to both direct and inverse theorems for trigonometric and algebraic polynomials.

Nonlinear approximation is dealt with only briefly in Chapter 6 where rational approximation is considered. Except for an interesting chapter on Hilbert's Thirteenth Problem of approximating a function of several variables by superpositions of functions of fewer variables, the book deals almost exclusively with univariate functions. A special feature is the inclusion of chapters on n-widths (a subject which has attracted considerable recent attention) and on entropy and capacity. This material is hard to find in book form, especially at this level.

I am happy to recommend this book to anyone who wants an introduction to Approximation Theory. The reader should be aware, however, that in addition to a considerable body of classical work which was not treated here, a tremendous amount of new work has been done in the past twenty years (especially in numerical methods, spline functions, and multivariate approximation). While the reader will have to look elsewhere for these more recent results, I am sure that he will find the present book an excellent foundation to build upon. We all owe a debt of gratitude to Chelsea for resurrecting this delightful little monograph.

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19[42-01, 42-04].—RONALD N. BRACEWELL, *The Hartley Transform*, Oxford Engineering Science Series, Vol. 19, Oxford Univ. Press, New York, 1986, vii + 160 pp., 24 cm. Price $24.95.

This book's stated purpose is to introduce and discuss applications of the Hartley transform and to compare it with the related Fourier transform.

After a brief introduction in Chapter 1, Chapter 2 defines the Hartley transform and shows how the Hartley transform coefficients relate to those of the Fourier transform. A number of examples are given pertaining to the calculation of the Hartley transform for a variety of problems. The power spectrum and the phase are shown to be readily expressible in terms of Hartley transform coefficients. Hartley and Fourier transform theorems are presented in Chapter 3, as well as relations between domains, while Chapter 4 presents a good discussion of discrete versus continuous transforms.

Much of the remainder of the book is devoted to the practical application of these transforms. Chapter 5 discusses digital filtering by convolution. Cyclic convolutions are discussed and contrasted with ordinary convolutions, and comparisons are made to Fourier convolutions. Examples are given involving low pass filtering and edge enhancement. The chapter ends with convolutions expressed in terms of matrix multiplications.

In Chapter 6, two-dimensional Hartley transforms are discussed; in Chapter 7, a description of a factorization technique is given and the transform is shown to be expressible as a matrix operation with the bulk of the chapter devoted to factorization of this matrix and the perturbation operations involved. Chapter 8 discusses details of a fast transform algorithm and various schemes to speed up all aspects of the calculation including rapid computation of trigonometric functions and methods for fast permutation. The concluding short Chapter 9 discusses optical Hartley transforms. A series of problems follows each chapter.

Appendix I gives a series of programs in BASIC for discrete Hartley transforms, while Appendix II presents an atlas of Hartley transforms.

This book presents a quite thorough discussion of Hartley transforms with special emphasis on the discrete transform. It is well written and gives the reader a number of applications of these transforms and detailed cases where computation via the Hartley transform has advantages over that done by more standard Fourier transform techniques.

There are a few minor typographical errors such as in the expression for the even part of the transform on page 11 and in the expression for the phase on page 18. In general, though, I found no errors of any consequence.